



# Effect of phase fluctuations of a laser on the dynamic process of an atom near a metallic layer

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Received 20 June 2006, accepted 15 December 2006

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**Abstract** : The effect of phase fluctuations of a laser on the atomic reflection of evanescent light is investigated. This effect is one of the main factors that limit the performance of the atomic mirror action obstructing the ultimate goal of near-perfect coherent reflection process. It is concluded that this effect plays a significant role hence should be incorporated in the treatment of quantum dynamics. By operating at low intensities, that could be achieved by the addition of thin metallic layer, these effects can only be minimized. Based on these considerations, the atomic mirror action is determined.

**Keywords** : Phase fluctuations, evanescent light, spontaneous force, dipole forces, optical potential

**PACS Nos.** : 03.75.Be, 32.90.+a

## 1. Introduction

There are two approaches to realize atom mirrors; reflection from surfaces, first demonstrated by Stern and his colleagues in 1929, marking the beginning of atom optics by demonstrating the reflection and diffraction of atoms from metallic and crystalline surfaces [1]. The second demonstration using an evanescent light field to reflect atoms which was first reported by Balykin *et al* [2] following a proposal by Cook and Hill [3]. Recently, there has been an extensive research in various quantum phenomena. This arose from controlling the atomic motion by the advent of tunable lasers. In part, this was influenced by the rapid progress in techniques to manipulate the trajectories of neutral atoms using light forces [4-7].

This paper focuses on the same mechanism of reflection that was proposed by Cook and Hill. This mechanism makes use of field evanescent into the vacuum region outside the planar surface of a dielectric block when the laser is internally reflected [8,9]. The

evanescent field sets up a repulsive dipole potential which acts on any neutral atom possessing a transition frequency at near resonance with, and positive detuned from, the laser frequency [10-13].

The evanescent field mechanism of atomic mirror has several advantages over the direct reflection from a surface. The evanescent field potential is purely repulsive, whereas the atom-surface potentials contain an attractive part as a result of the van der Waals potential that effectively acts at short distances ( $L \ll \lambda/4\pi$ ) and the Casimir-Polder at large distances ( $L \gg \lambda/4\pi$ ). Each of these potentials leads to large sticking probabilities [8-10,13]. In addition to that, the direct reflection requires some strict conditions of the surface quality in contrast to the evanescent mechanism that requires less demands on the surface. Moreover, the evanescent mechanism could be used to reflect atoms in internal states other than the ground state [14-17].

The main objective of this work is to investigate the effect of phase fluctuations of a laser source on atomic mirror action. It has been suggested that the effects of phase fluctuations can be reduced by decreasing the intensity of laser source, which can be achieved by using surface plasma [18,19]. Although, this technique has been successful to reduce the required intensity, the phase fluctuations can still play vital role on the dynamics of a two-level atom system. The intensity of laser source should not be reduced to such a level where van der Waals and Casimir-Polder interactions lead to sticking effect [8,9].

The effects of phase fluctuations are studied for the evanescent light due to the thin metallic layer. Bennett *et al* [9] have obtained significant results from the study of atomic mirror action induced by evanescent light due to the thin metallic layer. They have calculated the optical forces acting on the atom and have shown a mirror action involving the trajectories of reflecting atoms. They have also investigated the mirror action due to this type of system in low intensity regime ignoring the effect of phase fluctuations.

The present work incorporates the effect of phase fluctuations to obtain more accurate calculations. It is anticipated that the present treatment will help achieve the ultimate goal of obtaining a high degree of coherent atomic beams. There are some arguments that such level of coherence is basically a required condition in the study of atoms interferences [20]. In addition, the high degree of coherent atomic beams has received an impetus with the realization that such properties are needed to improve the experimental results in many applications of quantum phenomena [21,22].

The paper is organized as follows. Section 2 presents the main ingredients of our theoretical model which is a thin metallic layer deposited on the surface to provide an enhancement of the evanescent field and hence the repulsive potential. In Section 3, we present the basic formalism explaining the effect of phase fluctuations of a laser source on the optical forces. In Section 4, we show in detail the main factors that control the dynamic process in such atomic mirror system. Then, we evaluate the atomic mirror action due to evanescent mode under different levels of the phase fluctuations by considering the case of rubidium atom. Section 5 contains the comments and conclusions.

## 2. Theoretical model

The fundamental ingredients of the atomic mirror are illustrated in Figure 1. Here a metallic layer of thickness  $L$  is deposited on the planer surface of the dielectric substrate. Laser light of frequency  $\omega$  is incident at a specific angle and is propagating on the left hand side of the vertical axis within the dielectric substrate. This light is internally reflected at the inner interface between the dielectric substrate and the metallic layer, and partially leaks into the metallic layer.

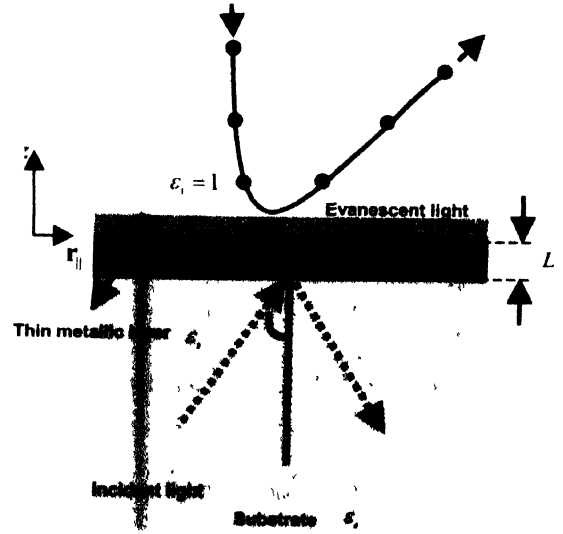


Figure 1. Schematic arrangement of an evanescent field atom mirror with a metallic layer. See the text for definition of the symbols.

The light in the metallic layer subsequently generates evanescent light in the vacuum region of the layer system. In presence of such light, a neutral atom possessing a transition  $\omega_0$  (where  $\omega_0 < \omega$ ) and incident due to gravity on the metallic layer would be subject to the field-induced forces. The field-induced forces include the light pressure force and the dipole force. The first force is responsible for the axial motion while the second force is responsible for the vertical motion. To evaluate field-induced forces, we should calculate the amplitude of the evanescent field. The calculation is carried out by applying the standard electromagnetic boundary conditions at the interface followed by normalizing the incident field. The electric field vector at frequency  $\omega$  can be written in quantized form as follows

$$\mathbf{E}(\mathbf{k}_{\parallel}, r, t) = \mathcal{F}(\mathbf{k}_{\parallel}, z) \exp i(\mathbf{k}_{\parallel}, r_{\parallel} - \omega t) a + H.C., \quad (1)$$

where  $a$  is a mode annihilation operator,  $H, C$ , stands for the Hermitian conjugate and  $\mathcal{F}(\mathbf{k}_{\parallel}, z)$  is the electric field distribution within the three regions of the layer system which can be written as :

$$\mathcal{F}(\mathbf{k}_{\parallel}, z < -L) = A \exp(ik_{z2}z) + B \exp(-ik_{z2}z), \quad (2)$$

$$\mathcal{F}(\mathbf{k}_{\parallel}, -L < z < 0) = C \exp(k_{z2}z) + D \exp(-k_{z2}z), \quad (3)$$

$$\mathcal{F}(\mathbf{k}_{\parallel}, z > 0) = G \exp(k_{z1}z). \quad (4)$$

In the above equations, the quantities  $A, B, C, D$  and  $G$  are field amplitude factors and  $\mathbf{k}_{\parallel}$  is the wave-vector parallel to the surface and its magnitude  $k_{\parallel}$  is given by

$$c^2 k_{\parallel}^2 = \omega^2 \varepsilon_2 \sin^2 \phi, \quad (5)$$

where  $\phi$  is the angle of incident. The sub-labeled 1, 2 and s refer to the substrate, the outer region and the metallic layer respectively. The real wave-vectors  $k_{z1}$ ,  $k_{z2}$  and  $k_{zs}$  can be defined as

$$c^2 k_{z1}^2 = c^2 k_{\parallel}^2 - \omega^2 \epsilon_1 > 0, \quad (6)$$

$$c^2 k_{z2}^2 = \omega^2 \epsilon_2 - c^2 k_{\parallel}^2 > 0, \quad (7)$$

$$c^2 k_{zs}^2 = c^2 k_{\parallel}^2 - \omega^2 \epsilon_s. \quad (8)$$

The dielectric function of the metallic layer  $\epsilon_s$  is given by

$$\epsilon_s = 1 - \frac{\omega_p^2}{\omega(\omega + i\delta)}, \quad (9)$$

where  $\omega_p$  is the plasma frequency of the metal which is defined as

$$\omega_p = \frac{n_0 e^2}{m^* \epsilon_0} \quad (10)$$

where  $m^*$  and  $e$  are the electronic effective mass and charge respectively,  $n_0$  is the volume electron density of the metal and  $\delta \ll \omega$  accounts for metallic layer plasma loss effects. By following the standard method of the quantization of the field, we can write the amplitude of the incident field in the unbounded bulk of material 2, as demonstrated by Bennett *et al* [9]

$$A^2 = \left( \frac{\hbar k_{z2}^2 c^2}{V \epsilon_0 \epsilon_2 \left( \omega \frac{\partial \epsilon_2}{\partial \omega} + 2\epsilon_2 \right) \omega} \right), \quad (11)$$

where  $V$  is a (large) quantisation volume of material 2.

### 3. The optical force with phase fluctuation

The optical forces associated with the near-resonant interaction of a laser light with atoms have been the subject of intensive theoretical and experimental study since the basic mechanism was first recognized. The simplest features can be described with reference to a two-level atom subject to an electromagnetic wave. In resonance, such an atom experiences two distinct forces : a dissipative force that arises from the absorption of the light by the atom followed by its spontaneous emission and a dipole force which arises from the non-uniformity of the field distribution. It is the combined influence of these forces and the gravity is responsible for the atomic mirror action. The dissipative force is

exploited for heating (and cooling) of the atomic motion whereas the dipole force is used for reflecting (or attracting). If we consider the effect of phase fluctuations, these forces in the steady state situation can be written as follow [23,24]

$$\langle F_{\text{spont}} \rangle = \frac{2\hbar\Gamma|\Omega|^2 \nabla \Theta}{\left[ \Gamma\gamma + \left( \Gamma\Delta^2/\gamma \right) + 2|\Omega|^2 \right]}, \quad (12)$$

and

$$\langle F_{\text{dip}} \rangle = -\frac{2\hbar\Gamma\Delta\Omega(\nabla\Omega)}{\gamma \left[ \Gamma\gamma + \left( \Gamma\Delta^2/\gamma \right) + 2|\Omega|^2 \right]}, \quad (13)$$

Since the dipole force is conservative, the corresponding potential is

$$U = \frac{\hbar\Gamma\Delta}{2\gamma} \ln \left[ 1 + \frac{2|\Omega|^2}{\Gamma\gamma + \left( \Gamma\Delta^2/\gamma \right)} \right], \quad (14)$$

where  $\gamma = \Gamma + \Gamma_\theta$  and  $\Gamma_\theta$  refer to the effect of phase fluctuations of a laser source. It is clear that the laser is considered perfectly coherent when  $\gamma = \Gamma$ , (i.e.  $\Gamma_\theta = 0$ ) and  $\Delta$  is the dynamic detuning given by

$$\Delta(\mathbf{v}) = \Delta_0 - \mathbf{k}_l \cdot \mathbf{v}, \quad (15)$$

where  $\Delta_0 = \omega - \omega_0$  is the static detuning of the light from the atomic resonance. The ability of the dipole force (or potential) to reflect (or attract) atom depends on the type of detuning [8,9]. It is very easy to recognize that  $F_{\text{dip}}$  will act as a repulsive force provided the detuning  $\Delta_0$  is positive and an attractive force when the detuning  $\Delta_0$  is negative.

We can easily determine the existence of phase fluctuations by separating the space dependence part from  $\Gamma$  and  $\Omega$ . This is done by introducing the definition  $\Gamma + \Gamma_0 \mathbf{R}$  and  $\Omega = \Omega_0 \mathbf{C}$ . By simple algebra we can find the ratio of the forces with fluctuations when present and when absent. Therefore, the conditions of the phase fluctuations effect on the two forces is given as [24]

$$F_{\text{spont}} \Rightarrow (\Delta/\Omega_0)^2 - \mathbf{R}(\rho + \mathbf{R}) > 0, \quad (16)$$

$$F_{\text{dip}} \Rightarrow \mathbf{R} \left[ 2\Omega^2 + (\Gamma_0/\Omega_0)^2 (\mathbf{R}\rho + 2\mathbf{R}^2) \right] < 0, \quad (17)$$

where the quantity  $\rho = (\Gamma_c/\Gamma_0)$  is a dimensionless parameter, which is the measure of phase fluctuations of the laser field. The condition (16) is easily satisfied for the usual range values of  $\Delta$  and  $\Omega_0$ , consequently the  $F_{\text{spont}}$  can be greater with fluctuations

present and then when they are absent. On the other hand, the condition (17) is not satisfied for the usual range values of  $\Gamma_0$  and  $\Omega_0$  except when  $R$  is negative which is never satisfied, consequently the  $F_{dip}$  with fluctuations can not exceed its value when they are absent. This clearly means that the a repulsive potential decreases with increase in fluctuations that decrease leads to reduce the activity of reflection process in atomic mirror system.

#### 4. Dynamic process

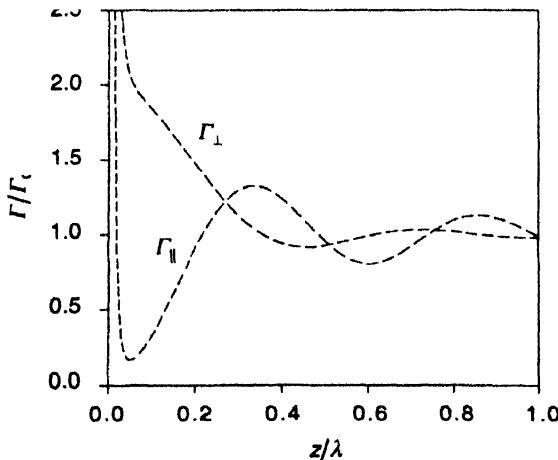
The whole dynamic process of optical forces in such system is controlled by three essential factors. Firstly, the modifications decay rate  $\Gamma$  of the dipole due to the vacuum field which can be given for the two possible orientations as follow [25]

$$\Gamma_{\parallel}(z) = \frac{3}{2} \Gamma_0 \left\{ \frac{3}{2} - \frac{\zeta \sin(2k_{1z}z + \delta)}{(2k_{1z}z)} - \frac{\zeta \cos(2k_{1z}z + \delta)}{(2k_{1z}z)^2} + \frac{\zeta \sin(2k_{1z}z + \delta)}{(2k_{1z}z)^3} \right\}, \quad (18)$$

$$\Gamma_z(z) = 3\Gamma_0 \left\{ \frac{1}{2} - \frac{\zeta \cos(2k_{1z}z + \delta)}{(2k_{1z}z)^2} + \frac{\zeta \sin(2k_{1z}z + \delta)}{(2k_{1z}z)^3} \right\}, \quad (19)$$

where  $\Gamma_0$  is the free-space dipole emission rate and  $\zeta$  and  $\delta$  are the additional reflection coefficient of the form  $\zeta \exp i\delta$  that is due to imperfect reflectivity of the metallic layer. Figure 2 shows the variation of the decay rate  $\Gamma_{\parallel}$  and  $\Gamma_z$  of an atom in front of a metallic layer with  $\zeta = 0.97$  and  $\delta = 0.008\omega_p^{silver}$  [19] relative to the rate in free-space against the distance from the surface relative to the free space wavelength of the dipole transition. At a distance where  $z \ll \lambda_0$ , both decay rates diverge while from  $z = \lambda_0/4$

the two decay rates oscillate about free space value until they are exactly fixed on  $\Gamma_0$  at  $z = \lambda_0$ .



**Figure 2.** Variations with distance  $z$  (in units of  $\lambda$ ) of the decay emission rate  $\Gamma_{\parallel}$  and  $\Gamma_z$  (in units of  $\Gamma_0$ ). See the text for the other parameters.

In contrast to the direct reflection of atomic beam from a surface, the reflection from potential induced by the evanescent wave reflects the atoms from some distance away from the surface. This distance can be evaluated from the decays exponentially of evanescent wave from the dielectric surface with characteristic length ( $L = 1/\beta$ ), which can be given as follows [26] :

$$\beta = \frac{2\pi}{\lambda_0} (\epsilon_1 \epsilon_s \sin \phi - 1)^{1/2}, \quad (20)$$

This means that the length of evanescence wave is governed by the angle of incidence  $\phi$ ,  $\epsilon_1$  and  $\epsilon_s$ . Therefore, this length can be controllable by a suitable selection of these parameters. In the atomic mirror, a short decay length ( $L \geq \lambda_0/2$ ) is desirable in order to reduce the effective laser-atom interaction time and hence the probability of the decay rates. Thus, the total decay rate  $\Gamma$  of such case can be taken to be the free-space value  $\Gamma_0$  and hence we do not need to include the effects of the field-dipole orientation picture [8,9]. This is, in fact, a very good approximation in the reflection process region which is far from the metallic surface. In addition to that, the effect of the attractive van der Waal's force can be completely neglected in such process because its effect appears only at distance ( $L \ll \lambda/4\pi$ ).

The second factor is the position-dependent Rabi frequency which characterizes the interaction of an atom with the electric fields. The square of the Rabi frequency can be given as demonstrated by Bennett *et al* [12] in the following form

$$\Omega^2(z \geq 0) = \Omega_0^2 \frac{4k_{z2}^2 c^2 (1 + k_{\parallel}^2 / k_{z1}^2) \exp(-2k_{z1}z)}{\epsilon_2 \left( \omega \frac{\partial \epsilon_2}{\partial \omega} + 2\epsilon \right) \omega^2 |\Theta|^2}, \quad (21)$$

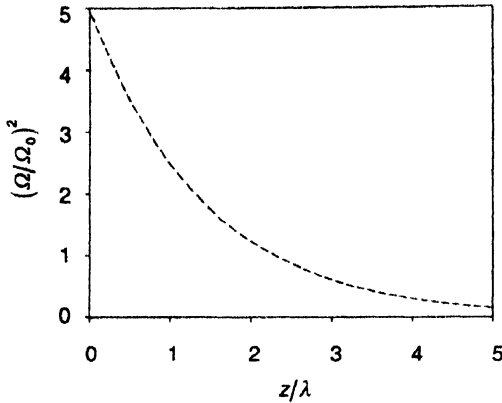
where the free-space Rabi frequency  $\Omega_0$  and the quantity  $\Theta$  is given respectively as

$$\Omega_0^2 = \left( \frac{I \mu^2}{2\hbar^2 \epsilon_0 c} \right), \quad (22)$$

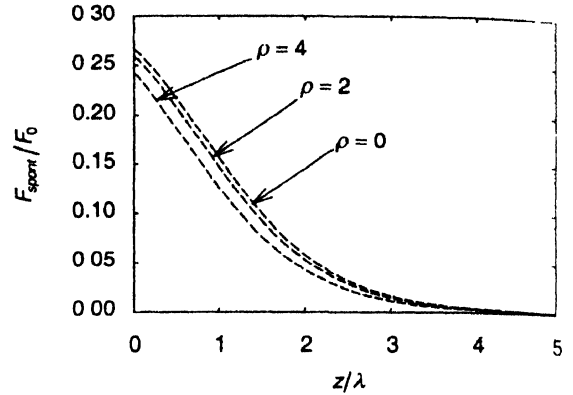
$$\Theta = \left\{ \left( 1 - i \frac{k_{z2}\epsilon_1}{k_{z1}\epsilon_2} \right) \cosh(k_{zs}L) + \left( \frac{k_{zs}\epsilon_1}{k_{z1}\epsilon_s} - i \frac{k_{z2}\epsilon_s}{k_{zs}\epsilon_2} \right) \sinh(k_{zs}L) \right\}, \quad (23)$$

Figure 3 displays the variation of the square Rabi frequency relative to the free-space value  $\Omega_0$  against the distance from the surface relative to the free space wavelength of the dipole transition of the rubidium atom ( $\lambda = 780nm$ ) with the angle of incidence of laser light fixed at  $\phi = 42.20^\circ$ . The parameter  $\delta$  entering the imaginary part of  $\epsilon_s$  is taken as  $\delta = 0.008\omega_p^{silver}$  and we have assumed that  $\epsilon_2$  does not depend on the frequency and so we set  $\partial \epsilon_2 / \partial \omega = 0$ . Of course, the pronounced repulsion effects would be expected in a condition corresponding to the peak of the Rabi frequency.

In Figure 4 we have shown the magnitude of the spontaneous force  $F_{spont}$  (relative to  $F_0 = 2\hbar k_{\parallel} \Gamma_0$ ) against  $z/\lambda$ . The curves correspond to different values of the degree of coherent  $\rho$ . From Figure 4, we conclude that with increase in fluctuations (i.e. as  $\rho$  increases), the maximum value of spontaneous force is reduced. This result is consistent with prediction of eq.(12).



**Figure 3.** Variations with distance  $z$  ( in units of  $\lambda$  ) of the Rabi frequency ( in units of  $\Omega_0$  ) See the text for the other parameters



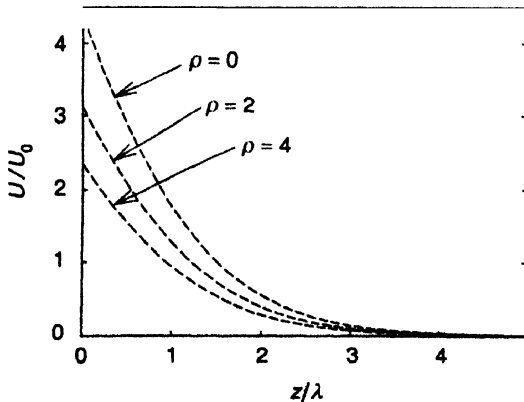
**Figure 4.** Variations with distance ( in units of  $\lambda$  ) of the spontaneous force acting on Rb atom ( in units of  $F_0$  ) See the text for the other parameters

In Figure 5, we have shown the dependence of the dipole potential  $U_{dip}$  (relative to  $U_0 = (1/2)\hbar\Gamma_0$ ) against  $z/\lambda$ . It can be clearly seen that the potential decreases as  $\rho$  increases. It evident from this figure that at low  $z/\lambda$ , there is a pronounced effect of the fluctuations on the potential. This indicates that a higher values of  $\rho$ , the atom may be attracted to the surface.

The trajectory of the atom of mass  $M$  approaching a mirror in a given set up is obtainable by solving the equation of motion

$$M \frac{dv}{dt} = F_{spont} + U_{dip} - Mg\hat{z}, \quad (24)$$

subject to given initial conditions. Figure 6 displays typical trajectories in the plane of incidence. As shown in the figure, the atom trajectories take different paths with any change of the degree of coherent  $\rho$ . Therefore, we can predict that, with the higher values of fluctuations, atoms will stick to the surface although the intensity is fixed.



**Figure 5.** Variations with distance  $z$  ( in units of  $\lambda$  ) of the dipole potential acting on Rb atom ( in units of  $U_0$  ). See the text for the other parameters.

## 5. Conclusion

The phase fluctuations of a laser source are considered one of the main factors that limit the performance of the reflection process due to the atomic mirror. It is true that these types of fluctuations would decrease at low intensities and the reduction of these effects



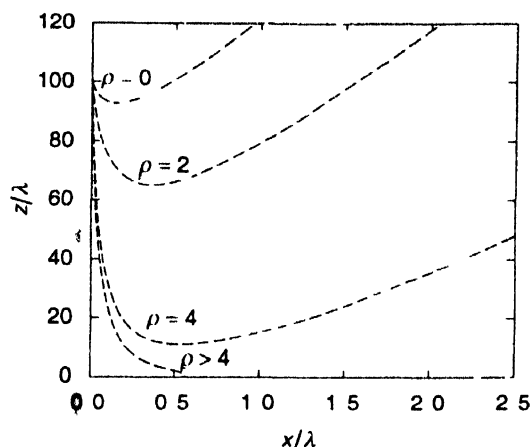
is the main reason behind investigation to devise an atom mirror operating at low intensities. One of the very successfully introduced way to enhance atomic mirror operating at low intensities was the mirror ingredients by the addition of thin metallic layer

We found that there was a considerable reduction in atomic mirrors activity due to these fluctuations even in such mirrors operating at low intensities. We have also observed that these fluctuations still play a very important role. The existence of these undesirable effects will make the production of atomic beams in a high degree of coherent is not available. Further reduction in the light intensity leads to attractive objective of near-perfect coherent reflection process. But, we should be careful, because there is a threshold in the laser intensity that should be determined to avoid the sticking effect of the atom due to the van der Waals and the Casimir-Polder interactions

In our calculations, we have not included the effects of the van der Waals and Casimir-Polder interactions on the atomic motion. The role of these interactions on atomic motion has already been clarified extensively both theoretically and experimentally [8-10,13]. In particular, they are known to be effective only at relatively very short distances from the surface

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**Figure 6.** The reflect of Rb atom due to a typical mirror arrangement with a metallic layer shown in Figure. The figure shows the atomic trajectory for an atom in the  $x - z$  plane of incidence with initial velocity components  $\theta_z = 0.5 \text{ m/s}$  and  $\theta_x = 0.7 \text{ m/s}$ . The initial position is  $x = 0$  and  $z = 600 \text{ nm}$ . The parameters are the same as those in Figures 4 and 5.

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